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BALLISTIC RESEARCH LABORATORY

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REPORT

RÖGGLA'S EQUATION AND ITS APPLICATION TO INTERIOR BALLISTIC PROBLEMS

(REVISED July 3, 1941)

by

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ADDENDUM TO BALLISTIC RESEARCH LABORATORY REPORT NO. 48 (REVISED)
by R. H. Kent

Initial and Maximum Values of the Roggla-Kent Dimensionless Pressure.

In Ballistic Research Laboratory Report No. 48 Kent derives Eq. (9), viz.

$$\eta \frac{d^3\eta}{d\theta^3} = \left(\frac{d^2\eta}{d\theta^2}\right)^r - \bar{\gamma}\ \frac{d\eta}{d\theta}\ \frac{d^2\eta}{d\theta^2},$$

where $\bar{\gamma}$ is a mean ratio of specific heats, r the exponent in the rate-of-burning law, η the free volume expansion ratio, and θ a dimensionless time variable. For fixed values of r and $\bar{\gamma}$ this is a universal differential equation applicable to all guns, subject to the limitations of the Roggla theory. For a universal value of the initial second derivative

$$\left(\frac{d^2\eta}{d\theta^2}\right) \equiv \underline{a}$$

it gives η as a universal function of θ, and thus $\frac{d^2\eta}{d\theta^2}$ as a universal function of η, viz. $\varphi(\eta)$. Except for the ordinate scale Roggla's P - η chart for his "zero degree of degressiveness" is a plot of $\varphi(\eta)$ vs η, so that $\frac{d^2\eta}{d\theta^2}$ may well be called the dimensionless pressure.

Together with the values r = 0.7 and $\bar{\gamma}$ = 1.2 Roggla used a value 1000/43.5 = 23.0 for the ratio of maximum pressure to initial pressure. Numerical integration of Eq. (9) led to the value 0.640 for $[\varphi(\eta)]_{max}$ for values of $\underline{a}$ about equal to 0.028. The value found for $[\varphi(\eta)]_{max}$ was not sensitive to the value of $\underline{a}$, being 0.640 for a = 0.03. Thus one can say that for a = 0.0278, $[\varphi(\eta)]_{max}$ has the value 0.640. The ratio of 0.640 to 0.0278, however, is just 23.0. We thus have the result that Roggla's "zero degressiveness" P - η curve corresponds to the values:

$$\varphi(1) = a = 0.0278$$
$$[\varphi(\eta)]_{max} = 0.640$$

The numerical integration showed the maximum value of $\varphi(\eta)$ to occur at η = 1.84, Roggla's value 1.93 thus being in error by 5%.

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RÖGGLA'S EQUATION AND ITS APPLICATION TO INTERIOR BALLISTIC PROBLEMS (REVISED)

Some time ago Röggla published a series of interior ballistic charts together with a pamphlet* explaining the method of computing velocities and pressures. While these charts have a useful field of application, the theoretical treatment of Röggla may also be used advantageously in calculating, without their aid, the performance of a new weapon when interior ballistic data for a weapon differing not too widely from it in caliber, muzzle velocity etc. are known or in calculating the effect of a change in bullet weight, web thickness etc. in a given weapon. For this reason an exposition is given of the deduction of Röggla's equation and a description of the method of its application to interior ballistic problems.

Röggla's equation is the following:

$$p_1 = p\left[\frac{(\bar{\gamma}_1-1)e_1\ \sigma_1 b_1}{(\bar{\gamma}-1)e\ \sigma b}\right]^{\frac{2}{3-2r}}\left(\frac{C_{o_1}}{C_o}\right)^{\frac{2}{3-2r}}\left(\frac{v_o}{v_{o_1}}\right)^{\frac{1}{3-2r}}\left(\frac{d_1}{d'}\right)^{\frac{1}{3-2r}}\left(\frac{a}{a_1}\right)^{\frac{2}{3-2r}}{}^{**}. \quad (1)$$

In this equation;

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p = pressure,

$\bar{\gamma}$ = a pseudo ratio of specific heats,

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e = the energy of the powder per unit weight,

* Neue Diagramme Für Die Angewandte Innere Ballistik, Theorie und Beispiele" Edmund Röggla, Pilsen, 1930

**Obtained from Röggla's (7) on page 6 of "Theorie und Beispiele."

RECORDED

σ = the density of the powder,

b = the specific rate of burning,

r = exponent of pressure in the rate of burning law,*

O_o = the initial surface of the charge,

v_o = the initial free volume,

G' = the reduced weight of the projectile which takes account also of the weight of charge,

q = the cross sectional area of the bore.

The derivation of Böggla's equation depends fundamentally upon Résal's equation which may be written in the form:

$$\overset{\text{I}}{e \sigma b O p^r dt} = \overset{\text{II}}{p\, dv} + \overset{\text{III}}{\frac{p\, dv + v\, dp}{\gamma - 1}} \qquad (2)$$

The term under I gives the amount of energy delivered by the powder charge in time dt on the assumption that the rate of burning is proportional to the rth power of the pressure. The term under II is the work done during the corresponding time.. The term under III represents the increase in the internal energy of the powder gas. However, it holds only on the contrary to fact assumption that the ratio of the specific heats is constant.

The internal energy of a gas of mass, M, is

$$M \int_0^T c_v dT = M \bar{c}_v T,$$

where c_v is the specific heat at constant volume and T the absolute temperature. In the case of most gases

* While the experiments of Crow and Grimshaw seem to have shown that when a pure organic powder is burnt, the rate of burning is directly proportional to the pressure, other experiments e.g. those in the 240 mm Howitzer tend to show that when there is a considerable admixture of black powder the exponent is reduced possibly to as low as .7.

Further work on the influence of black powder on the rate of burning needs to be done.

$$c_p - c_v = R,$$

where c_p is the specific heat at constant pressure and R is the gas constant. Hence we may write the internal energy of the gas as

$$M \bar{c}_v T = \frac{M \bar{c}_v R T}{c_p - c_v} = \frac{M p(v-b)*}{\frac{c_p}{c_v} - \frac{c_v}{c_v}} .$$

If c_v is a constant, then

$$\frac{c_p}{c_v} - 1 = \gamma - 1,$$

On the other hand, if c_v is not a constant we may designate

$$\frac{R}{c_v} \text{ by } (\bar{\gamma} - 1)$$

from which it follows that

$$\bar{\gamma} = 1 + \frac{R}{c_v} .$$

We call $\bar{\gamma}$ the pseudo ratio of specific heats.

If c_v is a function of temperature, then in general $\bar{\gamma}$ is also a function of temperature. If the variation in the temperature of the powder gas is not too great, $\bar{\gamma}$ may be taken as approximately constant while the projectile is in the bore. In view of this the γ in Résal's equation (2) should be replaced by $\bar{\gamma}$.

In the term under III, v really represents the free volume back of the projectile whereas under II, v represents total volume. By free volume is meant the volume back of the projectile minus the volume of the unburnt powder and the co-volume of the burnt powder. However, on the assumption that the co-volume of the powder is equal to the volume

* b is the co-volume in this expression.

of the equivalent mass of solid powder, the equation is still valid in this respect. While this is not accurately true, it is possibly not a serious error in view of the fact that Résal's equation should be used for only approximate computations.

We accordingly rewrite Résal's equation in place of (2) as

$$e \sigma b \, \theta \, p^r \, dt = p \, dv + \frac{p \, dv + v \, dp}{\bar{\gamma} - 1} \tag{2a}$$

and v now represents the free volume both under II and III.

We next introduce the free space ratio, η , which is the ratio of free volume at any time to the original free volume. We accordingly obtain

$$dv = d\eta \cdot v_o.$$

The assumption is made that the passive resistance is proportional to the pressure. As a result of this hypothesis, the work, $A = \int p dv$, is given by

$$A = \frac{G' c^2}{2g} ,$$

where G' is the reduced weight of the projectile, which takes account of kinetic energy of the powder gas and also of the work of the passive resistance and c is the velocity. It may readily be shown that if G is the weight of the projectile, L the weight of charge and if the pressure equivalent to the forcing resistance is given by

$$\frac{\mu}{gq} \cdot \frac{Gdc}{dt} , ^*$$

then the reduced weight of the projectile G' is given by

$$G' = G(1 + \mu + \frac{L}{3G}).$$

We accordingly also obtain

$$dA = \frac{G'}{g} cdc = pdv. \tag{3}$$

Since the velocity is the derivative of the travel with respect to the time the following relations are easily obtained:

* In other words, μ is the ratio of the pressure equivalent to the forcing resistance to the pressure effective for the acceleration of the projectile.

$$\frac{dc}{dt} = \frac{d^2s}{dt^2} = \frac{1}{q} v_o \frac{d^2\eta}{dt^2} \qquad (4)$$

$$p\, dv = \frac{G'}{g} \frac{v_o^2}{q^2} \frac{d\eta}{dt} \frac{d^2\eta}{dt^2} dt \qquad (5)$$

$$p = \frac{G'}{g} \frac{v_o}{q^2} \frac{d^2\eta}{dt^2}, dp = \frac{G'}{g} \frac{v_o}{q^2} \frac{d^3\eta}{dt^3} dt. \qquad (6)$$

By means of the substitution of these relations in (2a) one obtains

$$\frac{d^3\eta}{dt^3} = \dddot{\eta} = \frac{B\ \ddot{\eta}^r - \bar{\gamma}\dot{\eta}\ddot{\eta}}{\eta}{}^* , \qquad (7)$$

where $B = (\bar{\gamma}-1)\ e\,\sigma\, b\, O_o \left(\frac{g\, q^2}{G' v_o}\right)^{1-r} \cdot \frac{1}{v_o}$.

The above three paragraphs are practically equivalent to a translation of paragraph 2 of "Neue Diagramme für Die Angewandte Innere Ballistik, Theorie und Beispiele."

In our simplified treatment we assume that the total burning surface of the charge is constant, a condition which is approximately realized in the case of strip and tubular powder.

Thus Röggla's equation applies more precisely to grains with constant burning surface and not to degressively or to progressively burning grains. However, for the latter powders, it may be used as an approximation.

Let $\theta = jt$, so that $\dot{\eta} = j \frac{d\eta}{d\theta}$

$$\ddot{\eta} = j^2 \frac{d^2\eta}{d\theta^2}$$

$$\dddot{\eta} = j^3 \frac{d^3\eta}{d\theta^3} .$$

* The superscript dots signify differentiation with respect to time.

Then (7) becomes

$$j^3 \frac{d^3\eta}{d\theta^3} = \frac{B\, j^{2r}\left(\dfrac{d^2\eta}{d\theta^2}\right)^r - \bar{\gamma}\, j^3 \dfrac{d\eta}{d\theta}\, \dfrac{d^2\eta}{d\theta^2}}{\eta} \quad . \tag{8}$$

If we take $j = B^{\frac{1}{3-2r}}$, (8) reduces to

$$\eta \frac{d^3\eta}{d\theta^3} = \left(\frac{d^2\eta}{d\theta^2}\right)^r - \bar{\gamma} \frac{d\eta}{d\theta} \cdot \frac{d^2\eta}{d\theta^2} \quad . \tag{9}$$

It is thus evident that on Röggla's assumptions there is <u>one</u> differential equation for <u>all</u> interior ballistic trajectories.

We now take initial conditions as follows:

$$\eta_o = 1$$

$$\left(\frac{d\eta}{d\theta}\right)_o = 0$$

$$\left(\frac{d^2\eta}{d\theta^2}\right)_o = \underline{a}, \text{ a constant for all trajectories.}$$

The first of these conditions holds by definition, the second expresses the fact that the initial velocity is zero, while the third is undoubtedly contrary to fact but is very convenient and may not lead to excessive errors. There is only one solution, $\eta = f(\theta)$, of (9) satisfying these initial conditions and hence on the given assumptions there is only one interior ballistic trajectory, $\eta = f(\theta)$, for all combinations of gun, powder and projectile.

Equation (6) gives the pressure as a function of the time. Now

$$\ddot{\eta} = j^2 \frac{d^2\eta}{d\theta^2} = j^2 f''(\theta) = j^2 f''\left[f^{-1}(\eta)\right] = j^2 \varphi(\eta),$$

where the function f^{-1} is the inverse of f and $\varphi(\eta)$ is defined by

$$\varphi(\eta) \equiv f''\left[f^{-1}(\eta)\right] \quad .$$

Hence the pressure can be expressed as a function of the free space ratio, η, as follows:

$$p = \frac{G'}{g}\,\frac{v_o}{q^2}\,\ddot{\eta} = \frac{G'}{g}\,\frac{v_o}{q^2}\,j^2\,\varphi(\eta) = \frac{G'\,v_o\,B^{\frac{2}{3-2r}}}{g\,q^2}\,\varphi(\eta)$$

$$= \left[\frac{G'(\bar{\gamma}-1)^2\,e^2\,\sigma^2\,b^2\,p_o^2}{g\,q^2\,v_o}\right]^{\frac{1}{3-2r}}\varphi(\eta). \qquad (10)$$

The value of η at which $\varphi(\eta)$ is a maximum gives the position of maximum pressure.

If p and p_1 are the pressures for two different weapons, and η and η_1 their corresponding free space ratios, we have, letting $\frac{G'}{g}\,\frac{v_o}{q^2}$, the coefficient of $\ddot{\eta}$ in (5), be k and the corresponding coefficient for the case of the second weapon be k_1,

$$p = k\ddot{\eta}$$

$$p_1 = k_1\ddot{\eta}_1.$$

From the above relations we obtain

$$p = k\,j^2\,\frac{d^2\eta}{d\theta^2}$$

$$p_1 = k_1 j_1^2\,\frac{d^2\eta_1}{d\theta_1^2} \quad ,$$

where $\theta_1 = j_1 t$ and $j_1 = B_1^{\frac{1}{3-2r}}$.

and $\frac{P_1}{P} = \frac{k_1}{k}\left(\frac{B_1}{B}\right)^{\frac{2}{3-2r}} \quad \frac{\frac{d^2\eta_1}{d\theta_1^2}}{\frac{d^2\eta}{d\theta^2}}$.

For equal free space ratios, that is for $\eta_1 = \eta$,

$$\frac{P_1}{P} = \frac{k_1}{k}\left(\frac{B_1}{B}\right)^{\frac{2}{3-2r}} .$$

Substituting in this equation the values of k and B, we obtain Pöggla's equation

$$P_1 = P\left[\left(\frac{(\bar{\gamma}_1-1)c_1\sigma_1 b_1}{(\bar{\gamma}-1)c\sigma b}\right)^2\left(\frac{D_{o1}}{D_o}\right)^2\left(\frac{v_o}{v_{o1}}\right)\left(\frac{G_1'}{G'}\right)\left(\frac{G}{G_1}\right)^2\right]^{\frac{1}{3-2r}} \quad * \qquad (1)$$

Since for two given sets of conditions the ratio of P_1 to P is constant, it follows that the ratio of the starting pressures is the same constant. This is therefore an additional assumption. It may be stated in this form: the ratio of the starting pressures is assumed to be equal to the ratio of the maximum pressures.

It may easily be shown that if the shape of the grain and the density are unchanged

$$\frac{D_{o1}}{D_o} = \frac{C_1 w}{C w_1} \quad ** .$$

If the compositions of the powder are also identical, equation (1) becomes

* We distinguish between $\bar{\gamma}_1$ and $\bar{\gamma}$ in this expression is spite of our assumption that $\gamma_1 = \gamma$ because a relatively small change in $\bar{\gamma}$ will produce a relatively large change in $(\bar{\gamma} - 1)$.

** C is the weight of charge, w is the web thickness.

$$p_1 = p\left[\left(\frac{C_1 w}{C w_1}\right)^2 \left(\frac{v_o}{v_{o_1}}\right)\left(\frac{G_1'}{G'}\right)\left(\frac{c}{c_1}\right)^2\right]^{\frac{1}{3-2r}} . \qquad (11)$$

Several interesting deductions may be made directly from this equation; one is that if the web thickness decreases while the maximum pressure is kept the same the position of the point of maximum pressure will be at a greater distance from the breech than formerly.

It follows directly from equation (11) that if w_1 is less than w and the pressure remains unchanged, v_{o_1} must be greater than v_o. As is shown on page 7 the pressure is a function of the free space ratio. Hence if the initial free volume is increased it follows that the travel for a given free space ratio will be greater. Hence since the position of the point of maximum pressure depends directly on the free space ratio, it follows that for the quicker powder the point of maximum pressure will be further removed from the breech.

This equation may also be used for deriving the differential coefficients for the maximum pressure which are used in correcting maximum pressures for weight of projectile, chamber capacity, etc.

We write (See Hitchcock's "Differential Coefficients for Interior Ballistics", File B-II-23)

$$\frac{dp}{p} = \alpha_1 \frac{dC}{C} + \beta_1 \frac{dV}{V} + n_1 \frac{dG'}{G'} + \kappa_1 \frac{dw}{w} ,$$

where p represents the maximum pressure and V represents the chamber capacity.

If we now let

$$p_1 = p + dp,$$

$$C_1 = C + dC,$$

$$w_1 = w + dw, \text{ etc.},$$

we obtain from (11)

$$1 + \frac{dp}{p} = 1 + \frac{2}{3-2r} \cdot \frac{dC}{C} - \frac{1}{3-2r} \cdot \frac{dv_o}{v_o} + \frac{1}{3-2r} \cdot \frac{dG'}{G'} - \frac{2}{3-2r} \cdot \frac{dw}{w}. \quad (12)$$

In case the chamber capacity V is kept constant, then

$$dv_o = - \frac{dC}{\sigma} ,$$

since

$$v_o = V - \frac{C}{\sigma} .$$

In case the charge is kept constant while the chamber volume varies,

$$dv_o = dV.$$

Hence

$$dv_o = dV - \frac{dC}{\sigma}$$

and

$$\frac{dv_o}{v_o} = \frac{V}{v_o} \cdot \frac{dV}{V} - \frac{C}{\sigma v_o} \cdot \frac{dC}{C} .$$

Equation (12) for the differential coefficients thus becomes

$$\frac{dp}{p} = \frac{1}{3-2r} \left(2 + \frac{C}{\sigma v_o} \right) \frac{dC}{C} - \frac{1}{3-2r} \cdot \frac{V}{v_o} \cdot \frac{dV}{V} + \frac{1}{3-2r} \cdot \frac{dG'}{G'} - \frac{2}{3-2r} \frac{dw}{w} . \quad (13)$$

For purposes of comparison, the values of the differential coefficients have been computed for two guns from the above equation, taking r = .7, .8 and 1 and placed in a table together with the values given in Hitchcock's paper which are deduced from the Interior Ballistic Tables and also some experimental values.

In the Table the values computed by Röggla's equation are headed by R, Hitchcock's values by H, and the experimental values by E.

Gun	V	Wt. of Proj.	Wt. of Chg.	Exp.* r	α_1 R	α_1 H	α_1 E	β_1 R	β_1 H	β_1 E	n_1 R	n_1 H	κ_1 R	κ_1 H
75 mm Mod. 1897	84	11.72	1.316	.7	1.48			-.85			.625		-1.25	
				.8	1.69	1.43	2.01	-.98	-.75	-.92	.715	.63	-1.43	-1.33
				1.0	2.37			-1.37			1.00		-2.00	
3"M1 1917	296	15	4.61	.7	1.49			-.85			.625		-1.25	
				.8	1.70	1.57	1.67	-.98	-.77		.715	.65	-1.43	-1.46
				1.0	2.38			-1.37			1.00		-2.00	

NOTE: The few experimental values are obtained by the use of coppers, the coefficients might be appreciably different if they were determined by the piezo-electric gauge.

* The coefficients have been computed as shown above for various values of the exponent, r, of the pressure in the rate of burning law.

In conclusion we proceed to solve some problems by means of Röggla's equation and check them by the firing results.

1st Problem

Given

Chamber capacity - 84 cu. in.
Weight of charge - 1.316 lbs.
Density of powder - .057 lb/in^3
Weight of projectile - 11.72 lbs.
Maximum pressure - 23,350 lbs.

Required: to compute the maximum pressure in the same gun with the following conditions:

Chamber capacity - 84 cu. in.
Weight of charge - 1.551 lbs.
Weight of projectile - 15.96 lbs.
The powder is of the same lot.

The results obtained by the computations are the following:

For $r = 1$, $p_1 = 47,100$

For $r = .8$, $p_1 = 38,600$

For $r = .7$, $p_1 = 36,200$

Observed pressure, 35,200

2nd Problem

Given

$C = 20.2$ oz.
$V = 84$ cu. in.
$p = 30,370$ lb/in^2

Required: to compute p_1 if $C_1 = 18$ oz. and other variables are unchanged.

Result of computations:

$p_1 = 25,540$

p_1 observed = 25,300.

3rd Problem

Given

C = 1.26 lbs.
V = 34 cu. in.
p = 36,830 lb/in^2

Required: to compute p_1 for

C_1 = .445 lbs.

V_1 = 35 cu. in., and other variables unchanged.

The result of computation:

p_1 = 16,000 lb/in^2

p_1 observed = 16,150*.

From the above examples, it may be seen that very satisfactory results are obtained, using $r = .7$. Hence in spite of the fact that we know that the rate of burning of the pure organic powder in the closed chamber is unity, in view of the more satisfactory results obtained by the exponent .7, such an exponent should be used in applying Röggla's equation to ordinary problems. There is evidence that the reduction of the exponent below unity serves to compensate for the contrary to fact assumptions that the starting pressure is proportional to the maximum pressure and that the forcing resistance is proportional to the acceleration of the projectile.

I am indebted to Miss B. I. Hart for help in the revision.

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* The last two examples were computed by Mr. Lane, using $r = .7$.